Forrest

AP Physics C

How to use a differential equation in physics.

BY DEFINITION, a differential equation is a calculus function where the answer to the equation depends on how that same variable is behaving in the equation.

In other words, a differential equation relates the value of a function to the rate of change (or the derivative) of that same function.

Example:

Assume a ball of mass M is released from rest from the top of a tall building. It experiences a force of air resistance where $F = -kv^2$, where k is a positive constant. Assume the positive direction for all vector quantities is downward. Express all algebraic answers in terms of M, k and fundamental constants.

A.) Does the magnitude of the balls' acceleration increase, decrease or remain the same after its released?

____ Increase ____ Decrease ____ Stays the same.

Explain your answer

B.) Write (but do not solve) a differential equation for the instantaneous velocity V of the ball in terms of time t as it falls.

Think Newton's 2nd Law!

$F_{net} = m \cdot a \longrightarrow F_{net} = F_{grav} - F_{air}$

and $a = \frac{dv}{dt}$

$\Downarrow$

So....
$$\boxed{mg - kv^2 = M\frac{dv}{dt}}$$

C.) Determine the terminal velocity of the ball.

At terminal velocity, $a = 0\,m/s^2$, so $F_{net} = 0$ Newtons

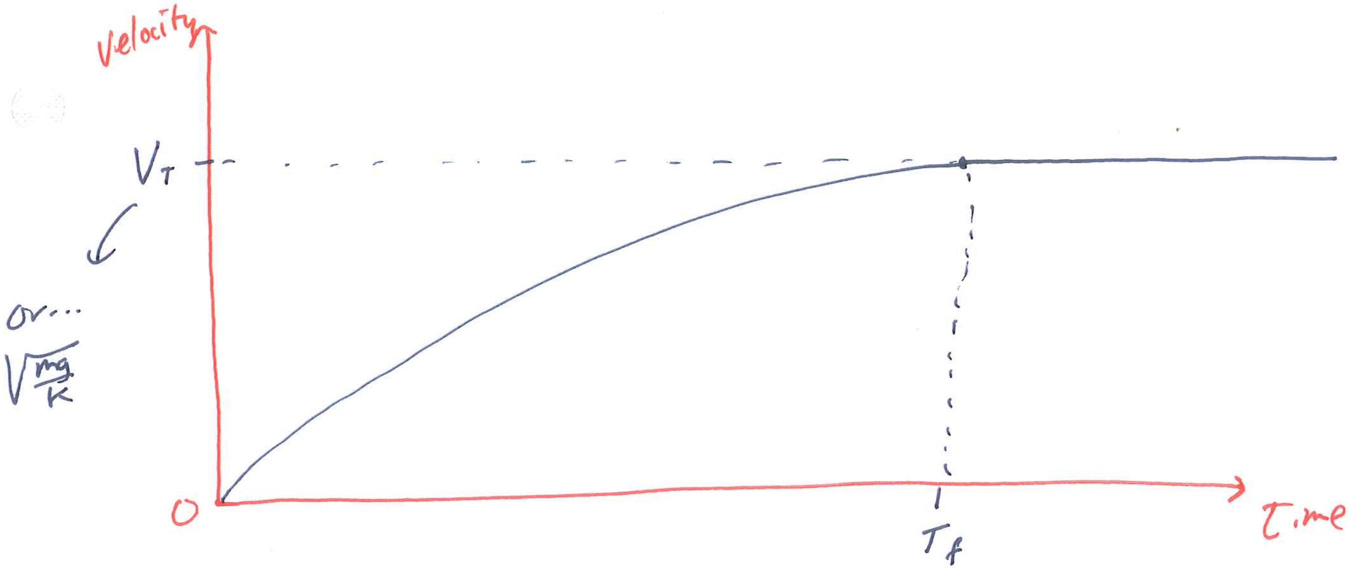
Therefore $\uparrow F_{air}$ $\downarrow F_{grav}$

So... $F_{grav} = F_{air}$ $\qquad V_T = V_{Terminal}$

$$mg = kV_T^2$$

$$\frac{mg}{k} = V_T^2 \; ; \quad \boxed{\sqrt{\frac{mg}{k}} = V_T}$$

D.) On the axes below, sketch a graph of velocity vs. time for the ball's flight until it reaches terminal velocity at time T_f.



As the ball gets closer and closer to the terminal velocity, the net force decreases, therefore the slope of the velocity-time graph should approach zero.