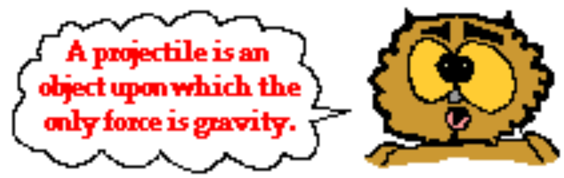
Motion and Forces in Two Dimensions – Lesson 2: Projectiles (parts a-e for your reading quiz) – This was copied from *The Physics Classroom*. No copyright infringement intended. For educational use only.

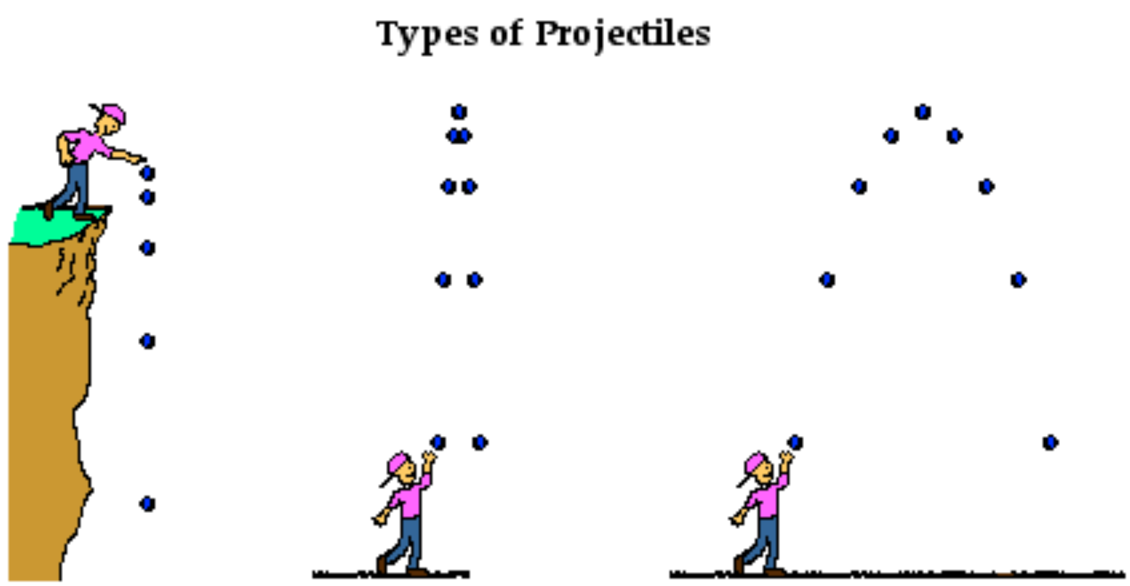
Lesson 2a) What is a Projectile?

In Unit 1 of the Physics Classroom Tutorial, we learned a variety of means to describe the 1-dimensional motion of objects. In Unit 2 of the Physics Classroom Tutorial, we learned how Newton's laws help to explain the motion (and specifically, the changes in the state of motion) of objects that are either at rest or moving in 1-dimension. Now in this unit we will apply both kinematic principles and Newton's laws of motion to understand and explain the motion of objects moving in two dimensions. The most common example of an object that is moving in *two dimensions* is a projectile. Thus, Lesson 2 of this unit is devoted to understanding the motion of projectiles.

Defining Projectiles



A projectile is an object upon which the only force acting is gravity. There are a variety of examples of projectiles. An object dropped from rest is a projectile (provided that the influence of air resistance is negligible). An object that is thrown vertically upward is also a projectile (provided that the influence of air resistance is negligible). And an object which is thrown upward at an angle to the horizontal is also a projectile (provided that the influence of air resistance is negligible). A projectile is any object that once *projected* or dropped continues in motion by its own inertia and is influenced only by the downward force of gravity.

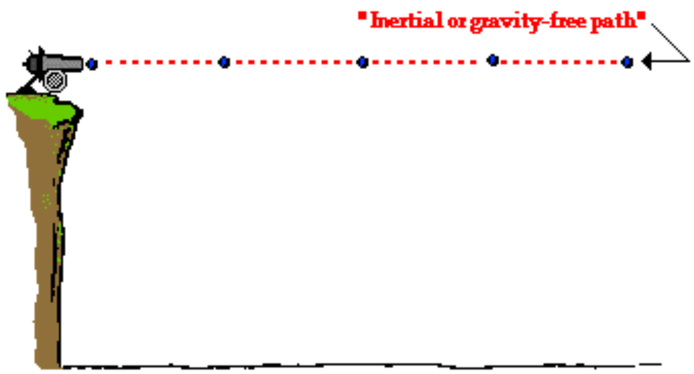


By definition, a projectile has a single force that acts upon it - the force of gravity. If there were any other force acting upon an object, then that object would not be a projectile. Thus, the free-body diagram of a projectile would show a single force acting downwards and labeled force of gravity (or simply F_{grav}). Regardless of whether a projectile is moving downwards, upwards, upwards and rightwards, or downwards and leftwards, the free-body diagram of the projectile is still as depicted in the diagram at the right. By definition, a projectile is any object upon which the only force is gravity.

Projectile Motion and Inertia

Many students have difficulty with the concept that the only force acting upon an upward moving projectile is gravity. Their conception of motion prompts them to think that if an object is moving upward, then there *must* be an upward force. And if an object is moving upward and rightward, there *must* be both an upward and rightward force. Their belief is that forces cause motion; and if there is an upward motion then there must be an upward force. They reason, "How in the world can an object be moving upward if the only force acting upon it is gravity?" Such students do not *believe* in Newtonian physics (or at least do not believe strongly in Newtonian physics). Newton's laws suggest that forces are only required to cause an acceleration (not a motion). Recall from the Unit 2 that Newton's laws stood in direct opposition to the common misconception that a force is required to keep an object in motion. This idea is simply not true! A force is not required to keep an object in motion. A force is only required to maintain an acceleration. And in the case of a projectile that is moving upward, there is a downward force and a downward acceleration. That is, the object is moving upward and slowing down.

To further ponder this concept of the downward force and a downward acceleration for a projectile, consider a cannonball shot horizontally from a very high cliff at a high speed. And suppose for a moment that the *gravity switch* could be *turned off* such that the cannonball would travel in the absence of gravity? What would the motion of such a cannonball be like? How could its motion be described? According to Newton's first law of motion, such a cannonball would continue in motion in a straight line at constant speed. If not acted upon by an unbalanced force, "an object in motion will ...". This is Newton's law of inertia.

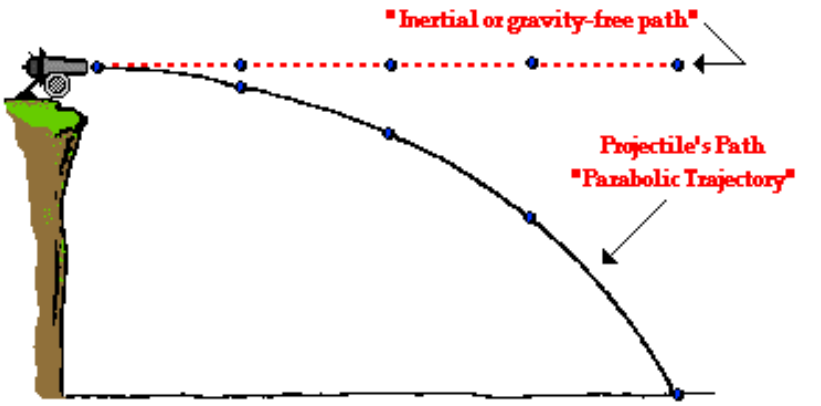


In the absence of gravity, an object in motion will continue in motion with the same speed and in the same direction.

Animation

Now suppose that the *gravity switch* is turned on and that the cannonball is projected horizontally from the top of the same cliff. What effect will gravity have upon the motion of the cannonball? Will gravity affect the cannonball's horizontal motion? Will the cannonball travel a greater (or shorter) horizontal distance due to the influence of gravity? The answer to both of these questions is "No!" Gravity will act downwards upon the cannonball to affect its vertical motion. Gravity causes a vertical acceleration. The ball will drop vertically below its otherwise straight-line, inertial path. Gravity is the downward force upon a projectile that influences its vertical motion and causes the parabolic trajectory that is characteristic of projectiles.

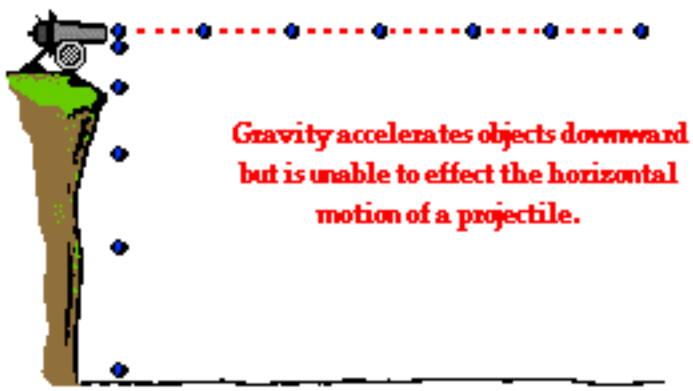
A projectile is an object upon which the only force is gravity. Gravity acts to influence the vertical motion of the projectile, thus causing a vertical acceleration. The horizontal motion of the projectile is the result of the tendency of any object in motion to remain in motion at constant velocity. Due to the absence of horizontal forces, a projectile remains in motion with a constant horizontal velocity. Horizontal forces are not required to keep a projectile moving horizontally. The only force acting upon a projectile is gravity!



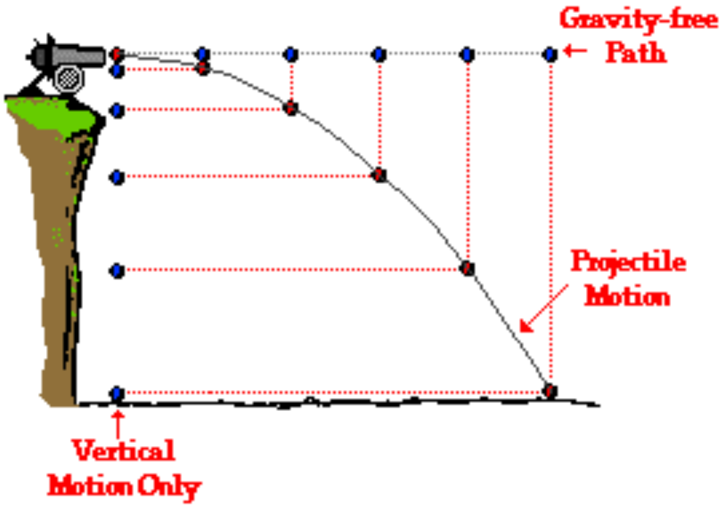
Lesson 2b) Characteristics of a Projectile's Trajectory

As discussed earlier in this lesson, a projectile is an object upon which the only force acting is gravity. Many projectiles not only undergo a vertical motion, but also undergo a horizontal motion. That is, as they move upward or downward they are also moving horizontally. There are the two components of the projectile's motion - horizontal and vertical motion. And since perpendicular components of motion are independent of each other, these two components of motion can (and must) be discussed separately. The goal of this part of the lesson is to discuss the horizontal and vertical components of a projectile's motion; specific attention will be given to the presence/absence of forces, accelerations, and velocity.

Horizontally Launched Projectiles

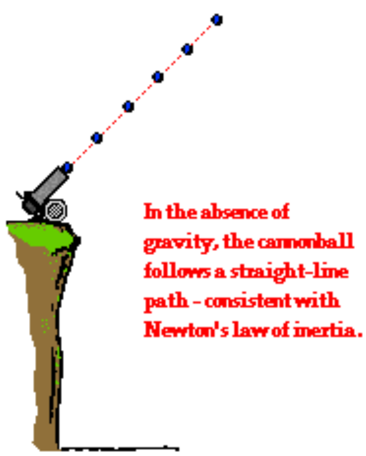


Let's return to our *thought experiment* from earlier in this lesson. Consider a cannonball projected horizontally by a cannon from the top of a very high cliff. In the absence of gravity, the cannonball would continue its horizontal motion at a constant velocity. This is consistent with the law of inertia. And furthermore, if merely dropped from rest in the presence of gravity, the cannonball would accelerate downward, gaining speed at a rate of 9.8 m/s every second. This is consistent with our conception of free-falling objects accelerating at a rate known as the acceleration of gravity.
If our thought experiment continues and we project the cannonball horizontally in the presence of gravity, then the cannonball would maintain the same horizontal motion as before - a constant horizontal velocity. Furthermore, the force of gravity will act upon the cannonball to cause the same vertical motion as before - a downward acceleration. The cannonball falls the same amount of distance as it did when it was merely dropped from rest (refer to diagram below). However, the presence of gravity does not affect the horizontal motion of the projectile. The force of gravity acts downward and is unable to alter the horizontal motion. There must be a horizontal force to cause a horizontal acceleration. (And we know that there is only a vertical force acting upon projectiles.) The vertical force acts perpendicular to the horizontal motion and will not affect it since perpendicular components of motion are independent of each other. Thus, the projectile travels with a constant horizontal velocity and a downward vertical acceleration.



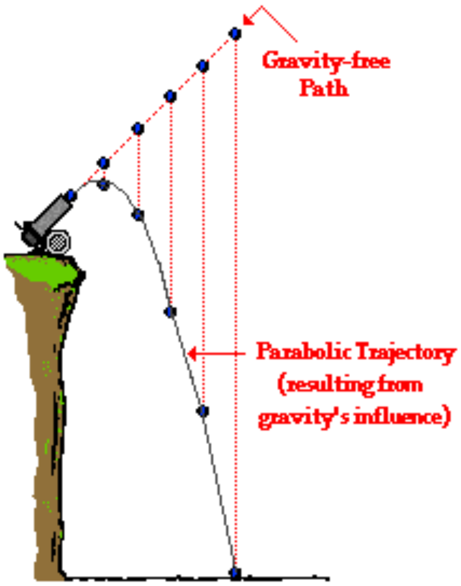
The above information can be summarized by the following table.

	Horizontal Motion	Vertical Motion	
Forces (Present? - Yes or No) (If present, what dir'n?)	No	Yes	The force of gravity acts downward
Acceleration (Present? - Yes or No) (If present, what dir'n?)	No	Yes	"g" is downward at 9.8 m/s/s
Velocity (Constant or Changing?)	Constant	Changing	(by 9.8 m/s each second)



Non-Horizontally Launched Projectiles

Now suppose that our cannon is aimed upward and shot at an angle to the horizontal from the same cliff. In the absence of gravity (i.e., supposing that the *gravity switch* could be *turned off*) the projectile would again travel along a straight-line, inertial path. An object in motion would continue in motion at a constant speed in the same direction if there is no unbalanced force. This is the case for an object moving through space in the absence of gravity. However, if the *gravity switch* could be *turned on* such that the cannonball is truly a projectile, then the object would once more *free-fall* below this straight-line, inertial path. In fact, the projectile would travel with a *parabolic trajectory*. The downward force of gravity would act upon the cannonball to cause the same vertical motion as before - a downward acceleration. The cannonball falls the same amount of distance in every second as it did when it was merely dropped from rest (refer to diagram below). Once more, the presence of gravity does not affect the horizontal motion of the projectile. The projectile still moves the same horizontal distance in each second of travel as it did when the *gravity switch* was turned off. The force of gravity is a vertical force and does not affect horizontal motion; perpendicular components of motion are independent of each other.



In conclusion, projectiles travel with a parabolic trajectory due to the fact that the downward force of gravity accelerates them downward from their otherwise straight-line, gravity-free trajectory. This downward force and acceleration results in a downward displacement from the position that the object would be if there were no gravity. The force of gravity does not affect the horizontal component of motion; a projectile maintains a constant horizontal velocity since there are no horizontal forces acting upon it.

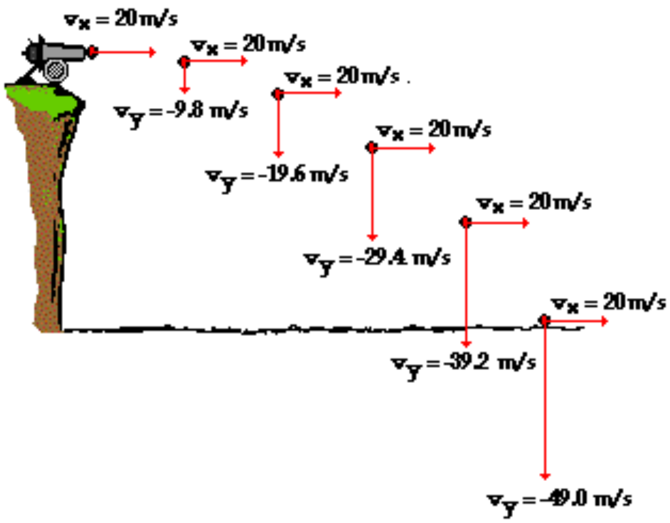
Lesson 2c) Describing Projectiles With Numbers: (Horizontal and Vertical Velocity)

So far in Lesson 2 you have learned the following conceptual notions about projectiles.

- A projectile is any object upon which the only force is gravity,
- Projectiles travel with a parabolic trajectory due to the influence of gravity,
- There are no horizontal forces acting upon projectiles and thus no horizontal acceleration,
- The horizontal velocity of a projectile is constant (a never changing in value),
- There is a vertical acceleration caused by gravity; its value is 9.8 m/s/s, down,
- The vertical velocity of a projectile changes by 9.8 m/s each second,
- The horizontal motion of a projectile is independent of its vertical motion.

In this portion of Lesson 2 you will learn how to describe the motion of projectiles numerically. You will learn how the numerical values of the x- and y-components of the velocity and displacement change with time (or remain constant). As you proceed through this part of Lesson 2, pay careful attention to how a conceptual understanding of projectiles translates into a numerical understanding.

Consider again the cannonball launched by a cannon from the top of a very high cliff. Suppose that the cannonball is launched horizontally with no upward angle whatsoever and with an initial speed of 20 m/s. If there were no gravity, the cannonball would continue in motion at 20 m/s in the horizontal direction. Yet in actuality, gravity causes the cannonball to accelerate downwards at a rate of 9.8 m/s/s. This means that the vertical velocity is changing by 9.8 m/s every second. If a vector diagram (showing the velocity of the cannonball at 1-second intervals of time) is used to represent how the x- and y-components of the velocity of the cannonball is changing with time, then x- and y- velocity vectors could be drawn and their magnitudes labeled. The lengths of the vector arrows are representative of the magnitudes of that quantity. Such a diagram is shown below.



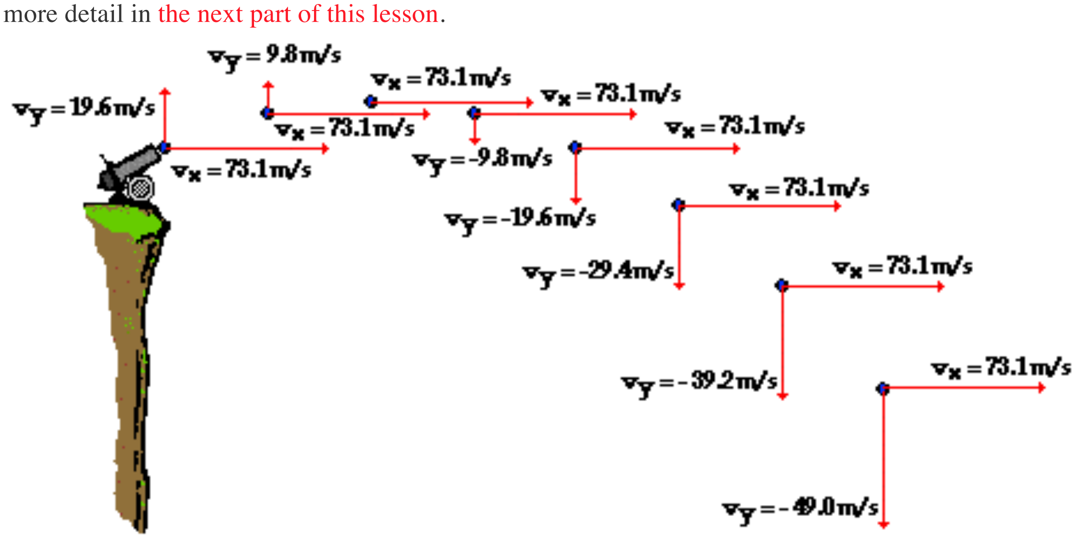
The important concept depicted in the above vector diagram is that the horizontal velocity remains constant during the course of the trajectory and the vertical velocity changes by 9.8 m/s every second. These same two concepts could be depicted by a table illustrating how the x- and y-component of the velocity vary with time.

Time	Horizontal Velocity	Vertical Velocity
0 s	20 m/s, right	0
1 s	20 m/s, right	9.8 m/s, down
2 s	20 m/s, right	19.6 m/s, down
3 s	20 m/s, right	29.4 m/s, down
4 s	20 m/s, right	39.2 m/s, down
5 s	20 m/s, right	49.0 m/s, down

The numerical information in both the diagram and the table above illustrate identical points - a projectile has a vertical acceleration of 9.8 m/s/s, downward and no horizontal acceleration. This is to say that the vertical velocity changes by 9.8 m/s each second and the horizontal velocity never changes. This is indeed consistent with the fact that there is a vertical force acting upon a projectile but no horizontal force. A vertical force causes a vertical acceleration - in this case, an acceleration of 9.8 m/s/s.

Animation

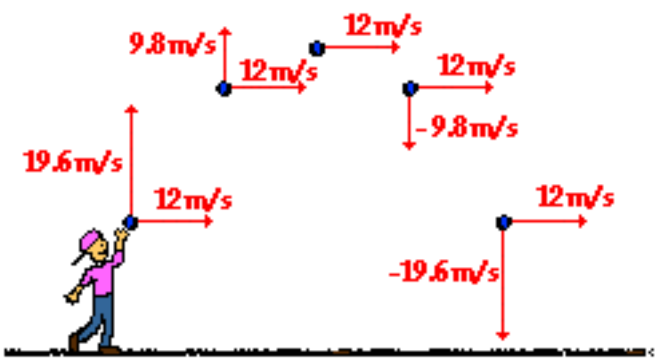
But what if the projectile is launched upward at an angle to the horizontal? How would the horizontal and vertical velocity values change with time? How would the numerical values differ from the previously shown diagram for a horizontally launched projectile? The diagram below reveals the answers to these questions. The diagram depicts an object launched upward with a velocity of 75.7 m/s at an angle of 15 degrees above the horizontal. For such an initial velocity, the object would initially be moving 19.6 m/s, upward and 73.1 m/s, rightward. These values are x- and y-components of the initial velocity and will be discussed in more detail in the next part of this lesson.



Again, the important concept depicted in the above diagram is that the horizontal velocity remains constant during the course of the trajectory and the vertical velocity changes by 9.8 m/s every second. These same two concepts could be depicted by a table illustrating how the x- and y-component of the velocity vary with time.

Time	Horizontal Velocity	Vertical Velocity
0 s	73.1 m/s, right	19.6 m/s, up
1 s	73.1 m/s, right	9.8 m/s, up
2 s	73.1 m/s, right	0 m/s
3 s	73.1 m/s, right	9.8 m/s, down
4 s	73.1 m/s, right	19.6 m/s, down
5 s	73.1 m/s, right	29.4 m/s, down
6 s	73.1 m/s, right	39.2 m/s, down
7 s	73.1 m/s, right	49.0 m/s, down

The numerical information in both the diagram and the table above further illustrate the two key principles of projectile motion - there is a horizontal velocity that is constant and a vertical velocity that changes by 9.8 m/s each second. As the projectile rises towards its peak, it is slowing down (19.6 m/s to 9.8 m/s to 0 m/s); and as it falls from its peak, it is speeding up (0 m/s to 9.8 m/s to 19.6 m/s to ...). Finally, the *symmetrical* nature of the projectile's motion can be seen in the diagram above: the vertical speed one second before reaching its peak is the same as the vertical speed one second after falling from its peak. The vertical speed two seconds before reaching its peak is the same as the vertical speed two seconds after falling from its peak. For non-horizontally launched projectiles, the direction of the velocity vector is sometimes considered + on the way up and - on the way down; yet the magnitude of the vertical velocity (i.e., vertical speed) is the same an equal interval of time on either side of its peak. At the peak itself, the vertical velocity is 0 m/s; the velocity vector is entirely horizontal at this point in the trajectory. These concepts are further illustrated by the diagram below for a non-horizontally launched projectile that lands at the same height as which it is launched.



The above diagrams, tables, and discussion pertain to how the horizontal and vertical components of the velocity vector change with time during the course of projectile's trajectory. Another vector quantity that can be discussed is the displacement. The numerical description of the displacement of a projectile is discussed in the next section of Lesson 2.

Lesson 2d) Describing Projectiles With Numbers: (Horizontal and Vertical Displacement)

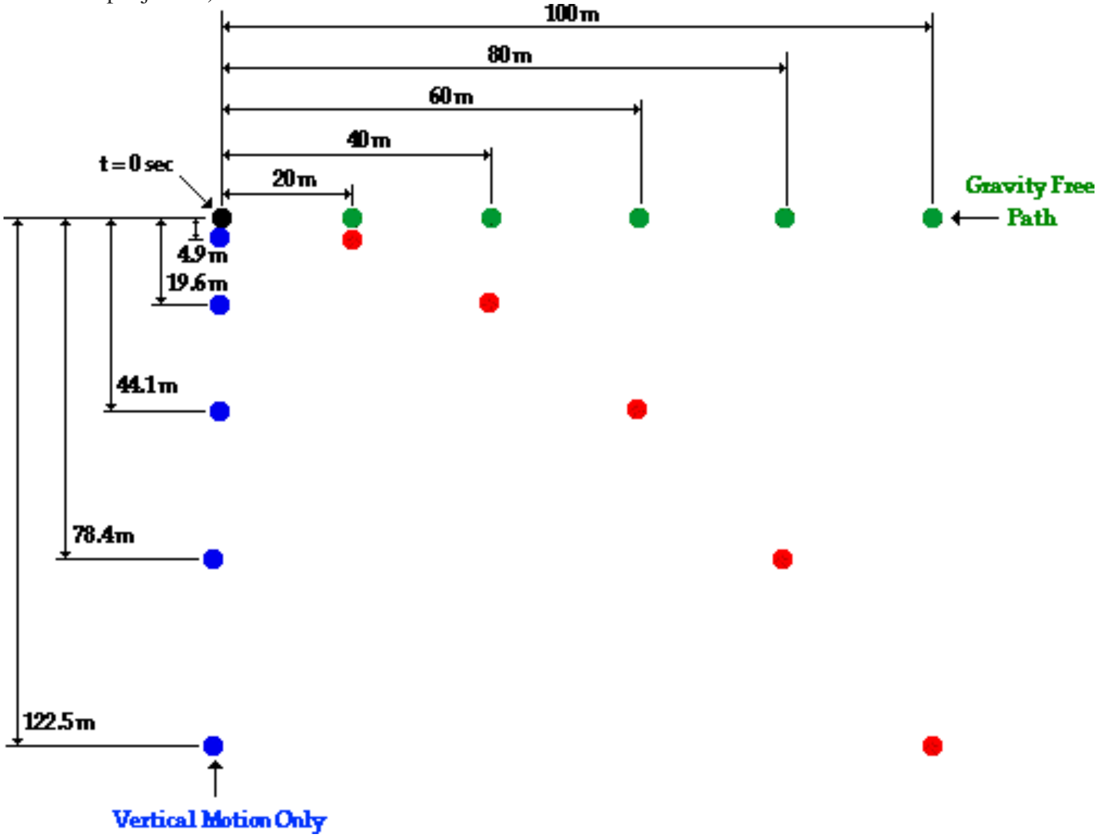
The previous diagrams, tables, and discussion pertain to how the horizontal and vertical components of the velocity vector change with time during the course of projectile's trajectory. Now we will investigate the manner in which the horizontal and vertical components of a projectile's displacement vary with time. As has already been discussed, the vertical displacement (denoted by the symbol *y* in the discussion below) of a projectile is dependent only upon the acceleration of gravity and not dependent upon the horizontal velocity. Thus, the vertical displacement (*y*) of a projectile can be predicted using the same equation used to find the displacement of a free-falling object undergoing one-dimensional motion. This equation was discussed in Unit 1 of The Physics Classroom. The equation can be written as follows.

$$y = 0.5 \bullet g \bullet t^2$$

(equation for vertical displacement for a horizontally launched projectile) where g is -9.8 m/s/s and t is the time in seconds. The above equation pertains to a projectile with no initial vertical velocity and as such predicts the vertical distance that a projectile falls if dropped from rest. It was also discussed earlier, that the force of gravity does not influence the horizontal motion of a projectile. The horizontal displacement of a projectile is only influenced by the speed at which it moves horizontally (v_{ix}) and the amount of time (t) that it has been moving horizontally. Thus, if the horizontal displacement (x) of a projectile were represented by an equation, then that equation would be written as

$$x = v_{ix} \bullet t$$

The diagram below shows the trajectory of a projectile (in red), the path of a projectile released from rest with no horizontal velocity (in blue) and the path of the same object when gravity is turned off (in green). The position of the object at 1-second intervals is shown. In this example, the initial horizontal velocity is 20 m/s and there is no initial vertical velocity (i.e., a case of a horizontally launched projectile).

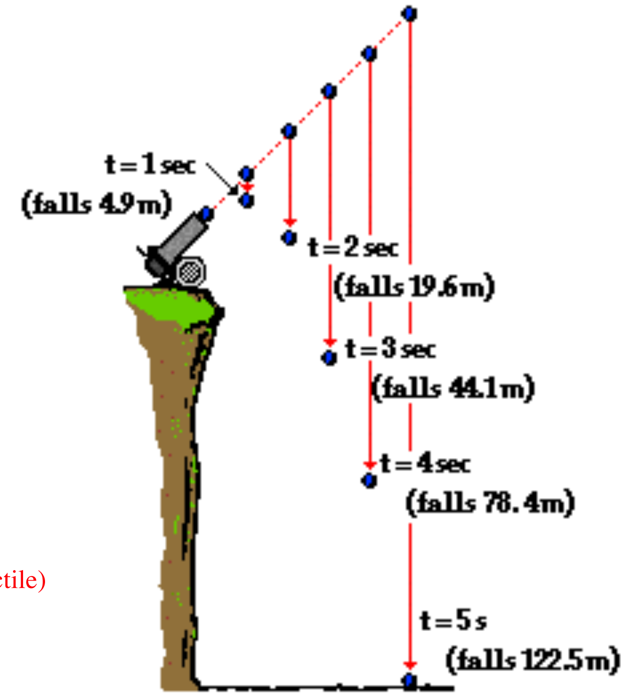


As can be seen in the diagram above, the vertical distance fallen from rest during each consecutive second is increasing (i.e., there is a vertical acceleration). It can also be seen that the vertical displacement follows the equation above ($y = 0.5 \bullet g \bullet t2$). Furthermore, since there is no horizontal acceleration, the horizontal distance traveled by the projectile each second is a constant value - the projectile travels a horizontal distance of 20 meters each second. This is consistent with the initial horizontal velocity of 20 m/s. Thus, the horizontal displacement is 20 m at 1 second, 40 meters at 2 seconds, 60 meters at 3 seconds, etc. This information is summarized in the table below.

Time	Horizontal Displacement	Vertical Displacement
0 s	0 m	0 m
1 s	20 m	-4.9 m
2 s	40 m	-19.6 m
3 s	60 m	-44.1 m
4 s	80m	-78.4 m
5 s	100 m	-122.5 m

Now consider displacement values for a projectile launched at an angle to the horizontal (i.e., a non-horizontally launched projectile). How will the presence of an initial vertical component of velocity affect the values for the displacement? The diagram below depicts the position of a projectile launched at an angle to the horizontal. The projectile still falls 4.9 m, 19.6 m, 44.1 m, and 78.4 m below the straight-line, gravity-free path. These distances are indicated on the diagram below (or on the next page).

The projectile still falls below its gravity-free path by a vertical distance of 0.5*g*t^2. However, the gravity-free path is no longer a horizontal line since the projectile is not launched horizontally. In the absence of gravity, a projectile would rise a vertical distance equivalent to the time multiplied by the vertical component of the initial velocity (viy• t). In the presence of gravity, it will fall a distance of $0.5 \cdot g \cdot t^2$. Combining these two influences upon the vertical displacement yields the following equation.



$$y = v_{iy} \cdot t + 0.5 \cdot g \cdot t^2$$

(equation for vertical displacement for an angled-launched projectile)

where v_{iy} is the initial vertical velocity in m/s, t is the time in seconds, and g = -9.8 m/s/s (an approximate value of the acceleration of gravity). If a projectile is launched with an initial vertical velocity of 19.6 m/s and an initial horizontal velocity of 33.9 m/s, then the x- and y- displacements of the projectile can be calculated using the equations above.

The following table lists the results of such calculations for the first four seconds of the projectile's motion.

Time	Horizontal Displacement	Vertical Displacement
0 s	0 m	0 m
1 s	33.9 m	14.7 m
2 s	67.8 m	19.6 m
3 s	101.7 m	14.7 m
4 s	135.6 m	0 m

The data in the table above show the symmetrical nature of a projectile's trajectory. The vertical displacement of a projectile *t* seconds before reaching the peak is the same as the vertical displacement of a projectile *t* seconds after reaching the peak. For example, the projectile reaches its peak at a time of 2 seconds; the vertical displacement is the same at 1 second (1 s before reaching the peak) is the same as it is at 3 seconds (1 s after reaching its peak). Furthermore, the time to reach the peak (2 seconds) is the same as the time to fall from its peak (2 seconds).

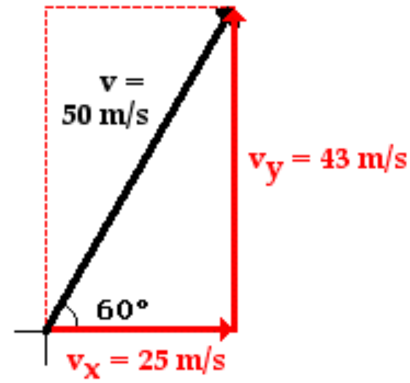
Lesson 2e) Initial Velocity Components

It has already been stated and thoroughly discussed that the horizontal and vertical motions of a projectile are independent of each other. The horizontal velocity of a projectile does not affect how far (or how fast) a projectile falls vertically. Perpendicular components of motion are independent of each other. Thus, an analysis of the motion of a projectile demands that the two components of motion are analyzed independent of each other, being careful not to *mix* horizontal motion information with vertical motion information. That is, if analyzing the motion to determine the vertical displacement, one would use kinematic equations with vertical motion parameters (initial *vertical* velocity, final *vertical* velocity, *vertical* acceleration) and not horizontal motion parameters (initial horizontal velocity, final horizontal velocity, horizontal acceleration). It is for this reason that one of the initial steps of a projectile motion problem is to determine the components of the initial velocity.

Determining the Components of a Velocity Vector

Earlier in this unit, the method of vector resolution was discussed. Vector resolution is the method of taking a single vector at an angle and separating it into two perpendicular parts. The two parts of a vector are known as components and describe the influence of that vector in a single direction. If a projectile is launched at an angle to the horizontal, then the initial velocity of the projectile has both a horizontal and a vertical component. The horizontal velocity component (v_x) describes the influence of the velocity in displacing the projectile horizontally. The vertical velocity component (v_y) describes the influence of the velocity in displacing the projectile vertically. Thus, the analysis of projectile motion problems begins by using the trigonometric methods discussed earlier to determine the horizontal and vertical components of the initial velocity.

Consider a projectile launched with an initial velocity of 50 m/s at an angle of 60 degrees above the horizontal. Such a projectile begins its motion with a horizontal velocity of 25 m/s and a vertical velocity of 43 m/s. These are known as the horizontal and vertical components of the initial velocity. These numerical values were determined by constructing a sketch of the velocity vector with the given direction and then using trigonometric functions to determine the sides of the *velocity* triangle. The sketch is shown at the right and the use of trigonometric functions to determine the magnitudes is shown below. (If necessary, review this method on an earlier page in this unit.)



$$\cos 60° = \frac{v_x}{50 \text{ m/s}}$$

$$v_x = 50 \text{ m/s} * \cos 60°$$

$$v_x = 25 \text{ m/s}$$

$$\sin 60° = \frac{v_y}{50 \text{ m/s}}$$

$$v_y = 50 \text{ m/s} * \sin 60°$$

$$v_y = 43 \text{ m/s}$$

Try Some More!

Need more practice? Use the Velocity Components for a Projectile *widget* on the Physics Classroom website!

As mentioned above, the point of resolving an initial velocity vector into its two components is to use the values of these two components to analyze a projectile's motion and determine such parameters as the horizontal displacement, the vertical displacement, the final vertical velocity, the time to reach the peak of the trajectory, the time to fall to the ground, etc. This process is demonstrated on the remainder of this page. We will begin with the determination of the time.

Determination of the Time of Flight

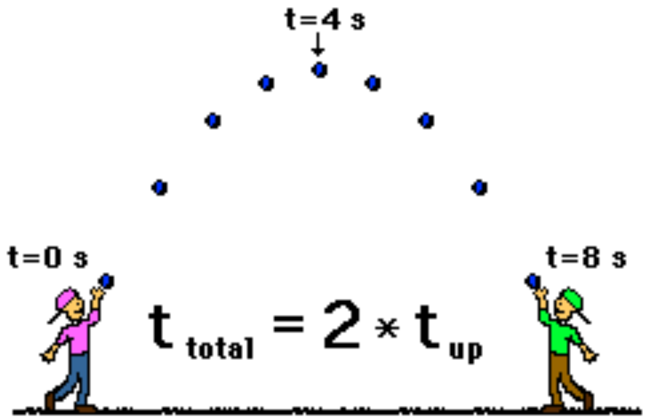
The time for a projectile to rise vertically to its peak (as well as the time to fall from the peak) is dependent upon vertical motion parameters. The process of rising vertically to the peak of a trajectory is a vertical motion and is thus dependent upon the initial vertical velocity and the vertical acceleration (g = 9.8 m/s/s, down). The process of determining the time to rise to the peak is an easy process - provided that you have a solid grasp of the concept of acceleration. When first introduced, it was said that acceleration is the rate at which the velocity of an object changes. An acceleration value indicates the amount of velocity change in a given interval of time. To say that a projectile has a vertical acceleration of -9.8 m/s/s is to say that the vertical velocity changes by 9.8 m/s (in the - or downward direction) each second. For example, if a projectile is moving upwards with a velocity of 39.2 m/s at 0 seconds, then its velocity will be 29.4 m/s after 1 second, 19.6 m/s after 2 seconds, 9.8 m/s after 3 seconds, and 0 m/s after 4 seconds. For such a projectile with an initial vertical velocity of 39.2 m/s, it would take 4 seconds for it to reach the peak where its vertical velocity is 0 m/s. With this notion in mind, it is evident that the time for a projectile to rise to its peak is a matter of dividing the vertical component of the initial velocity (v_{iy}) by the acceleration of gravity.

Time	Vertical Velocity
0 s	39.2 m/s, up
1 s	29.4 m/s, up
2 s	19.6 m/s, up
3 s	9.8 m/s, up
4 s	0 m/s

A projectile with an initial vertical velocity of 39.2 m/s requires 4 seconds to rise to the peak of its trajectory.

$$t_{up} = \frac{v_{iy}}{g}$$

Once the time to rise to the peak of the trajectory is known, the total time of flight can be determined. For a projectile that lands at the same height which it started, the total time of flight is twice the time to rise to the peak. Recall from the last section of Lesson 2 that the trajectory of a projectile is symmetrical about the peak. That is, if it takes 4 seconds to rise to the peak, then it will take 4 seconds to fall from the peak; the total time of flight is 8 seconds. The time of flight of a projectile is twice the time to rise to the peak.



If it takes a projectile 4 seconds to rise to its peak, then it will take a total of 8 seconds to move through the air from start to finish.

Determination of Horizontal Displacement

The horizontal displacement of a projectile is dependent upon the horizontal component of the initial velocity. As discussed in the previous part of this lesson, the horizontal displacement of a projectile can be determined using the equation

$$x = v_{ix} \bullet t$$

If a projectile has a time of flight of 8 seconds and a horizontal velocity of 20 m/s, then the horizontal displacement is 160 meters (20 m/s • 8 s). If a projectile has a time of flight of 8 seconds and a horizontal velocity of 34 m/s, then the projectile has a horizontal displacement of 272 meters (34 m/s • 8 s). The horizontal displacement is dependent upon the only horizontal parameter that exists for projectiles - the horizontal velocity (v_{ix}).

Determination of the Peak Height

A non-horizontally launched projectile with an initial vertical velocity of 39.2 m/s will reach its peak in 4 seconds. The process of rising to the peak is a vertical motion and is again dependent upon vertical motion parameters (the initial vertical velocity and the vertical acceleration). The height of the projectile at this peak position can be determined using the equation

$$y = v_{iy} \bullet t + 0.5 \bullet g \bullet t^2$$

where v_{iy} is the initial vertical velocity in m/s, g is the acceleration of gravity (-9.8 m/s/s) and t is the time in seconds it takes to reach the peak. This equation can be successfully used to determine the vertical displacement of the projectile through the first half of its trajectory (i.e., peak height) provided that the algebra is properly performed and the proper values are substituted for the given variables. Special attention should be given to the facts that the t in the equation is the time up to the peak and the g has a negative value of -9.8 m/s/s.